Biskra University
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NUMERICAL ANALYSIS OF ENGINEERING STRUCTURES BY ADVANCED FINITE ELEMENTS

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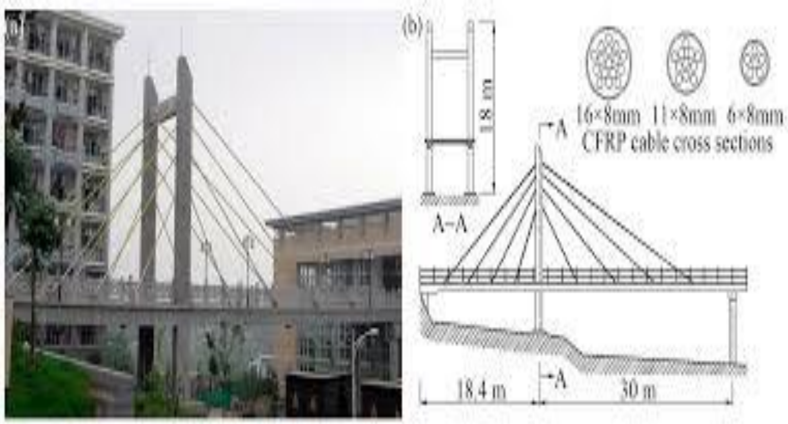
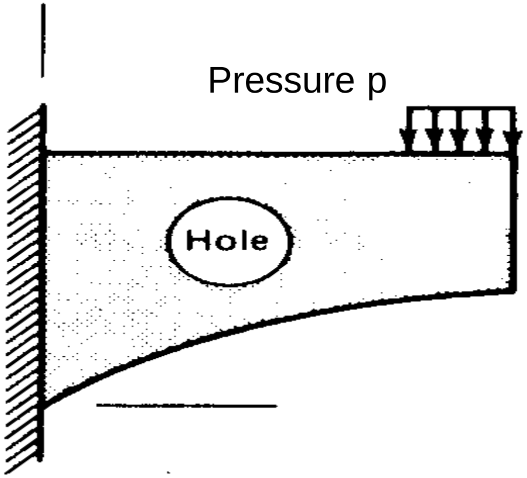
Engineering Structures

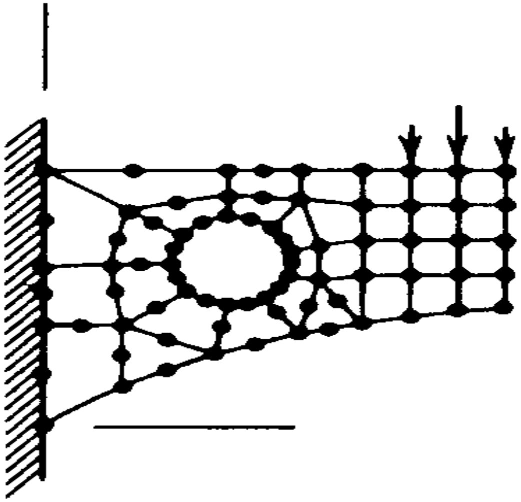


Modelling Structures

Various considerations:

- Type of elements • Size of elements • Location of nodes
- Local features in the structure (Stress concentrations)
- Meshing of the body • Material properties • External loads B.C.





A plan structure of arbitrary shape - A possible finite element model of the structure

- **Different Formulations (FEM)** .
- Displacement model - Stress model - Mixed model - Hybrid model, Strain Based Approach

Advantageous of S.B.A.

S.B.A Direct interpolation--------→ Better displacements (**Integration**)
Displacement Approach --------→ **Derivation.**
Easy satisfaction of the main two convergence criteria (constant strains and rigid body movement).
Possibility of enriching displacements field by terms of high order without the introduction of intermediate nodes or of supplementary degrees of freedom (**allowing so to treat the problem of locking**).

Description Of The Strain Based Approach

Example:

Procedure for the development of displacement field (S.B.A) Rectangular plane elasticity element

$$\varepsilon_x = U_{,x} = (\partial U/\partial x)$$

$$\varepsilon_y = V_{,y} = (\partial V/\partial y)$$

$$\gamma_{xy} = U_{,y} + V_{,x} = (\partial U/\partial y) + (\partial V/\partial x)$$

Final displacement functions:

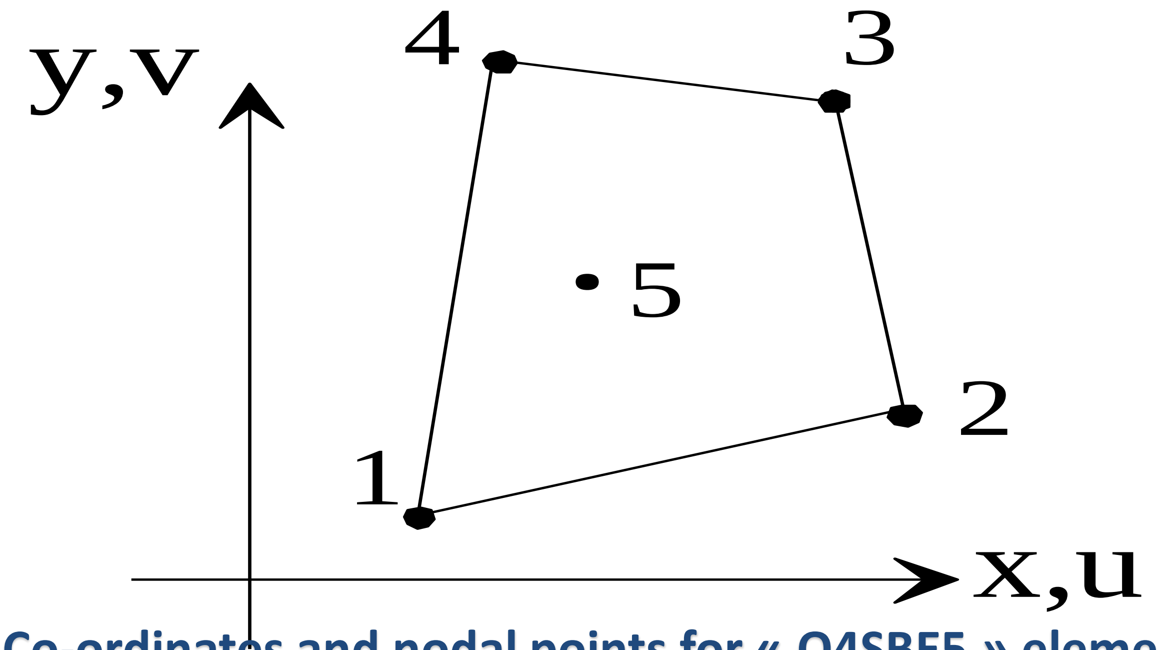
$$U = a_1 - a_3 y + a_4 x + a_5 xy - a_7 \frac{y^2}{2} + a_8 \frac{y}{2}$$

$$V = a_2 + a_3 x + a_6 y - a_5 \frac{x^2}{2} + a_7 xy + a_8 \frac{x}{2}$$

Finite elements Based on the Strain Approach

Membrane elements

Quadrilateral Membrane Element Q4SBE5 (Hamadi et al. 2016)



Co-ordinates and nodal points for « Q4SBE5 » element

Displacement fields

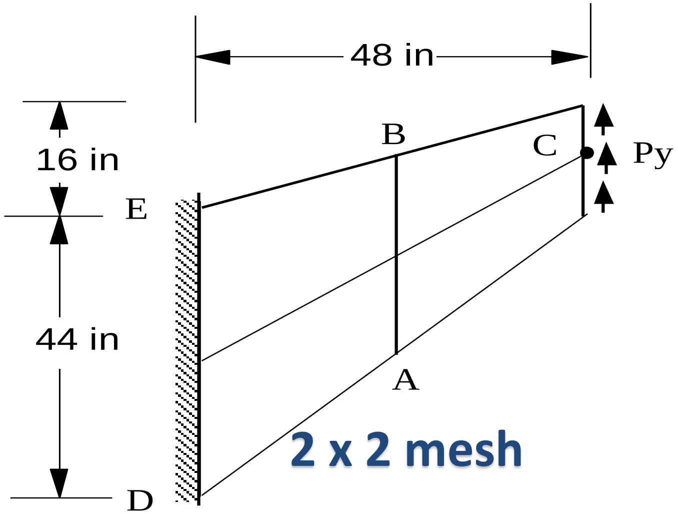
$$U = a_1 - a_3 y + a_4 x + a_5 xy - a_7 y^2 (R+1)/2 + a_8 y/2 + a_9 (x^2 - Hy^2)/2$$

$$V = a_2 + a_3 x - a_5 x^2 (R+1)/2 + a_6 y + a_7 xy + a_8 x/2 + a_{10} (y^2 - Hx^2)/2$$

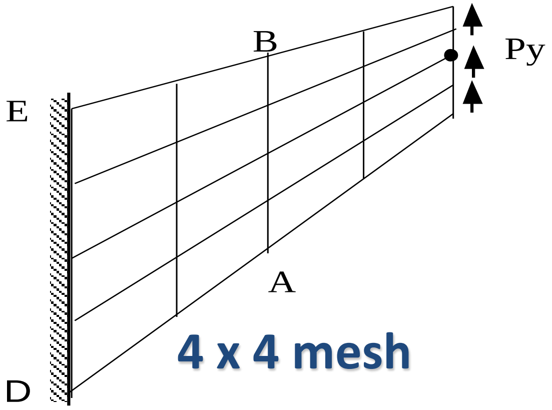
$$H = \frac{2}{(1-\nu)} \qquad R = \frac{2\nu}{(1-\nu)}$$

Application 1: Tapered Panel under End shear

This problem is proposed by Cook as a test for the accuracy of quadrilateral elements and Bergan et al.



2 x 2 mesh



4 x 4 mesh

Py = 1 pi (uniformly distributed load)

E = 1 psi, ν = 1/3 Thickness t =1 in

Boundary conditions:

U = V = 0 (DE)

Tapered panel subjected to end shear; data and meshes

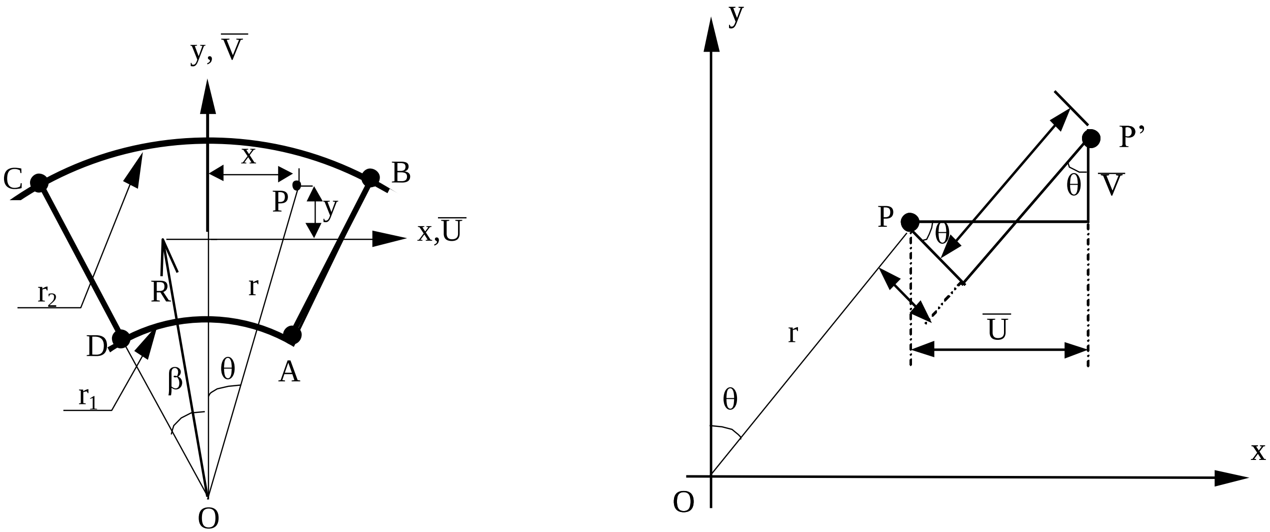
Normalised results for tapered panel under end shear

Element	2 x 2 mesh			4 x 4 mesh		
	V_C	σ_{maxA}	σ_{minB}	V_C	σ_{maxA}	σ_{minB}
Q4	0,496	0,437	0,533	0,766	0,756	0,719
AQ	0,890	0,780	0,900	0,965	0,936	1,010
Allman	0,848	0,771	0,856	0,953	0,956	0,997
MAQ	0,890	0,779	0,886	0,965	0,941	0,967
QR4b	0,941	0,879	1,059	0,980	0,990	0,997
Bergan	0,852	0,720	0,898	0,938	0,902	0,849
Q4SBE5	1,0652	1,508	1,171	1,011	1,004	0,992
32 x 32 mesh Cook	1,000 (23,90)	1,000 (0,236)	1,000 (-0,201)	1,000 (23,90)	1,000 (0,236)	1,000 (-0,201)

Comments:

The results obtained for the deflection and principal stresses for the refined mesh (4x4) are very good compared to an accurate solution given by Bergan and Felippa using a (32x32) mesh (error 1 %).

Sector Element SBMS-BH (Polar Coordinates)



(a) (b)

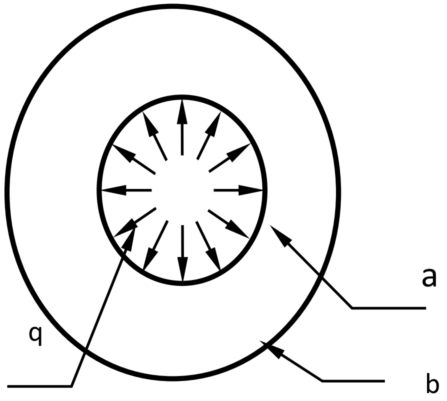
Coordinates systems and displacements for the sector element

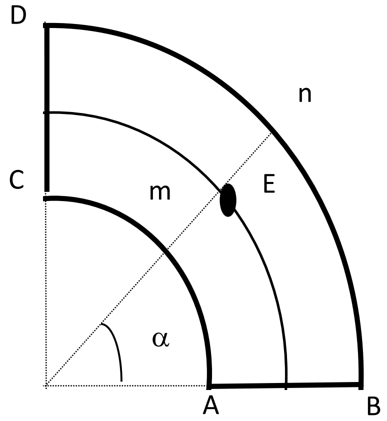
Description and displacement field

$$U = a_1 - a_3\, y + a_4\, x + a_5\, xy - 0.5\, a_7\, y^2 + 0.5\, a_8\, y + 0.5\, a_9\, x^2$$

$$V = a_2 + a_3\, x - 0.5\, a_5\, x^2 + a_6\, y + a_7\, xy + 0.5\, a_8\, x + 0.5\, a_{10}\, y^2$$

Application 2: Thick cylinder under internal pressure





Thick cylinder under internal pressure

Geometrical and material properties:

Internal radius a = 20 mm Thickness t = 1 mm

External radius b = 40 mm Poisson ratio $\nu = 0,3$

Young's modulus $E = 2\ 10^5$ MPa (Steel),

$s_e = 210$ MPa $\alpha = \pi/4$

Condition of symmetry:

AB and CD $V_{\square} = 0$

Internal pressure

$q = 0,1\ KN/mm^2$

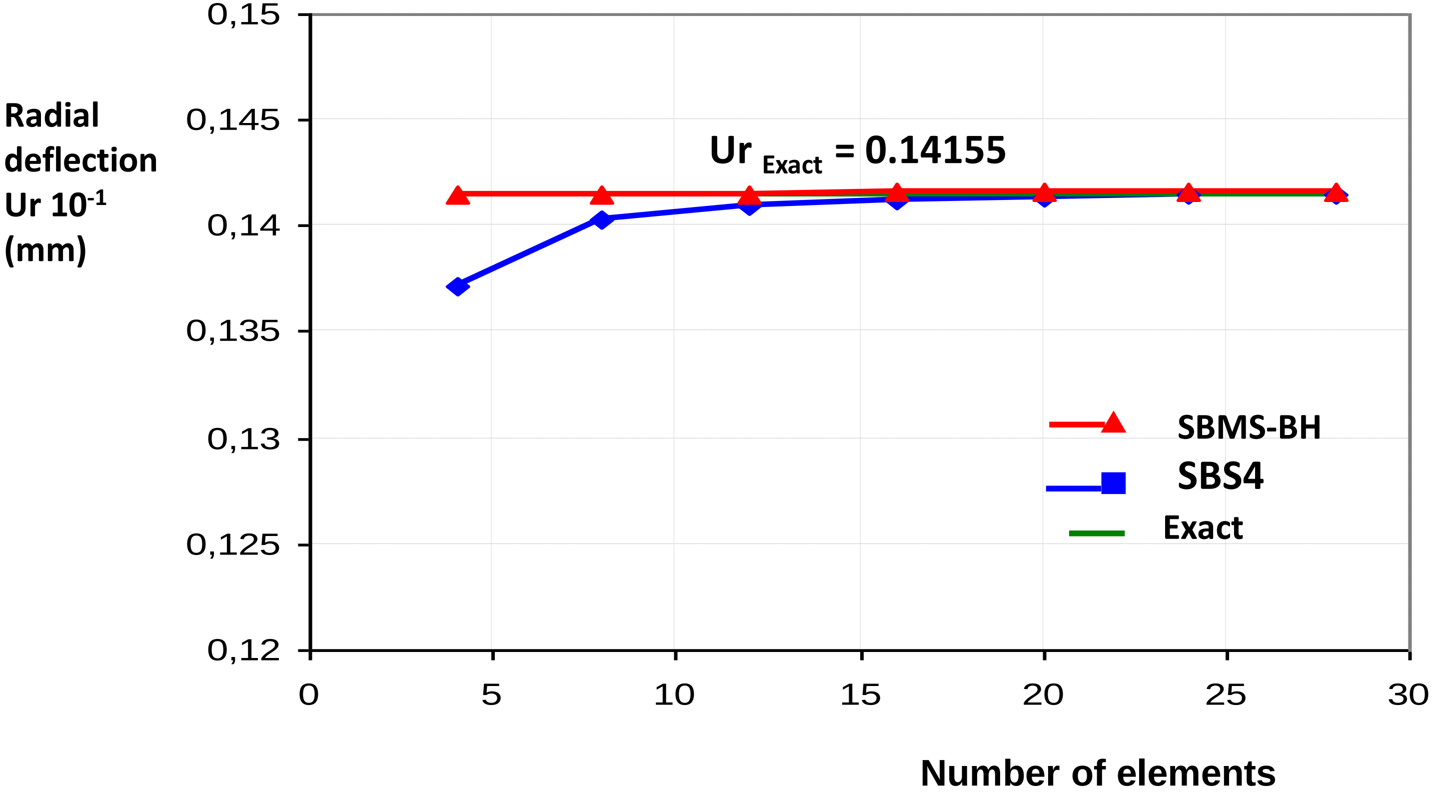
The results obtained at point E (r = 30 mm) :
Ur: Radial deflections Ur, The stresses and ,

$$U_r = \frac{(1+\nu)}{E(b^2 - a^2)}\left[(1-2\nu)(a^2 P_i - b^2 P_e)r + \frac{a^2 b^2}{r}(P_i - P_e)\right]$$

$$V_\theta = 0$$

$$\sigma_r = \frac{1}{(b^2 - a^2)}\left[a^2 P_i - b^2 P_e + \frac{a^2 b^2}{r^2}(P_e - P_i)\right]$$

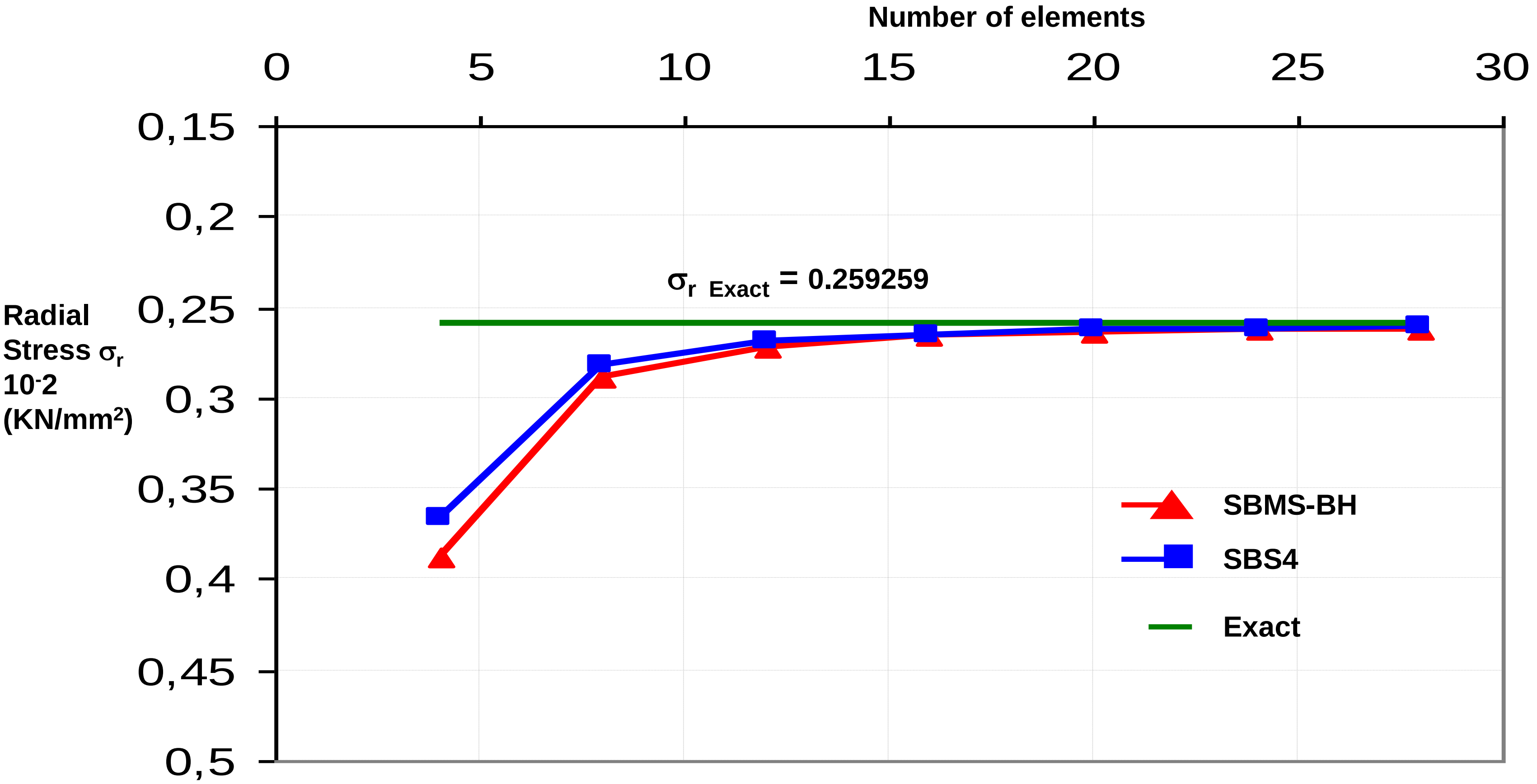
$$\sigma_\theta = \frac{1}{(b^2 - a^2)}\left[a^2 P_i - b^2 P_e - \frac{a^2 b^2}{r^2}(P_e - P_i)\right]$$



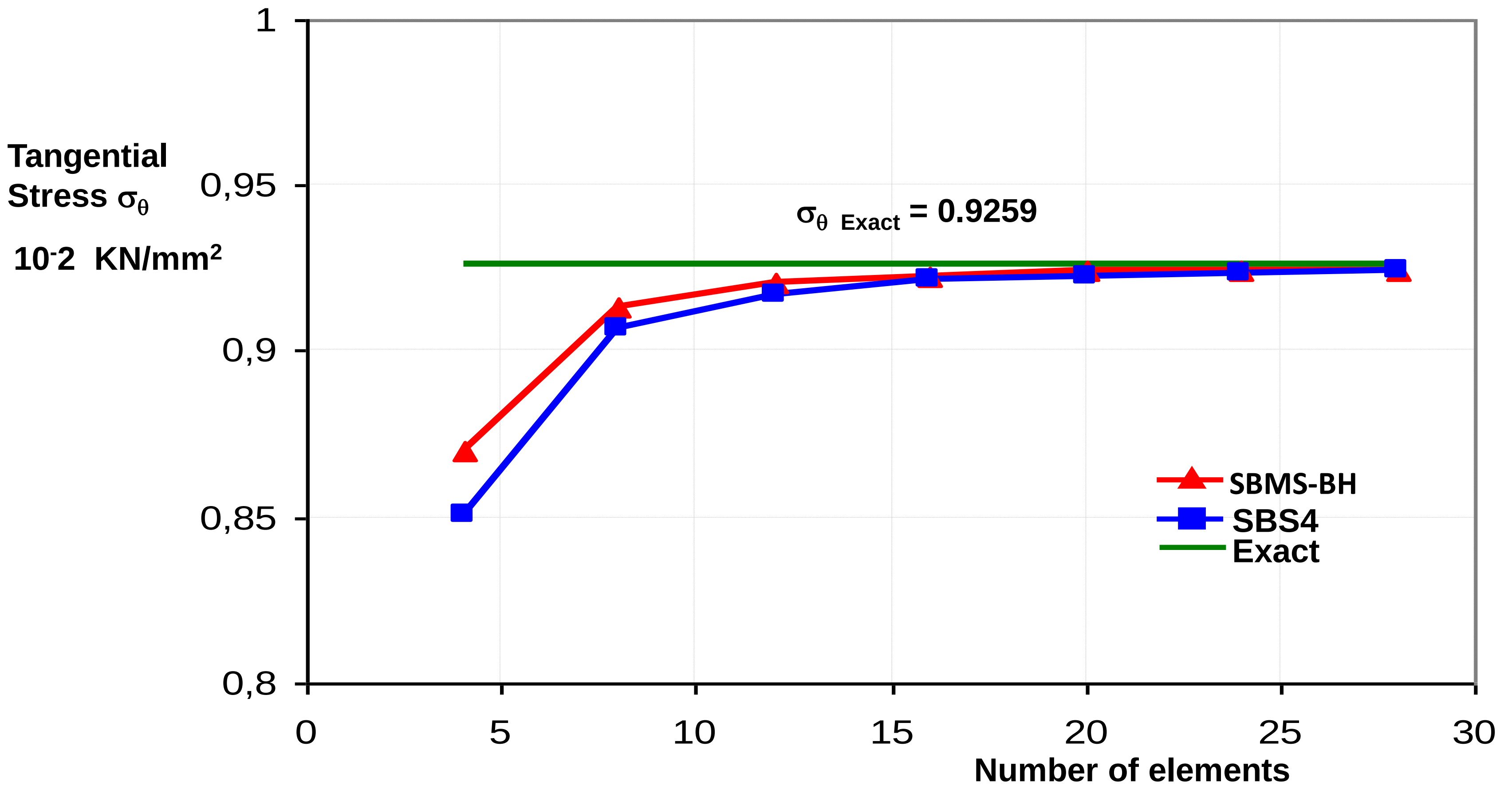
Convergence curve for the radial deflection U_r at point E (r =30 mm)

Comments:

High degree of accuracy obtained with element SBMS-BH
(the error accounts = 0.063 % of the exact solution with 2x2 meshes only).

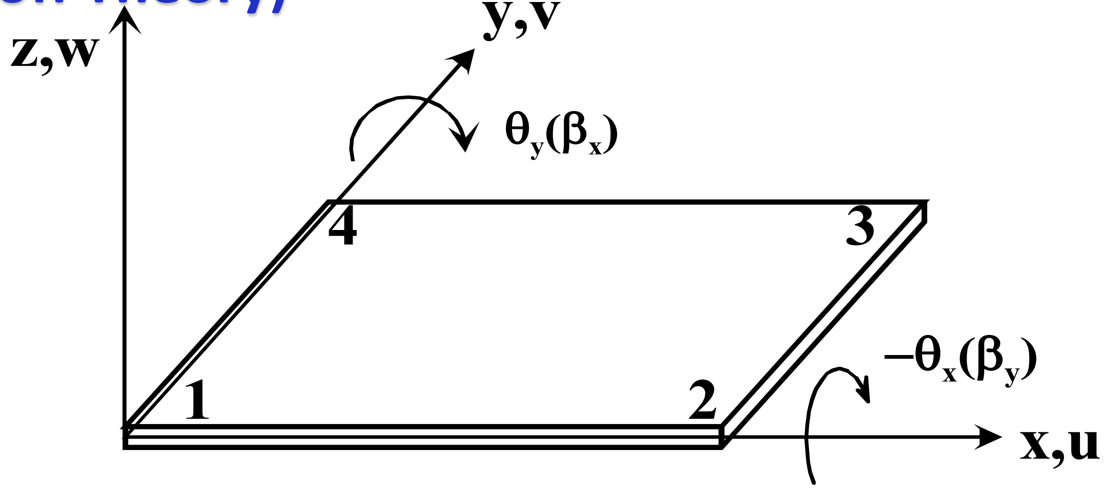


Convergence curve for the radial Stress σ_r at point E (r = 30 mm



Convergence curve for the tangential Stress σ_θ at point E (r = 30 mm)

Rectangular Plate Bending Element Based On the Strain Approach "SBRPK" (Kirchhoff Theory)



The displacement fields:

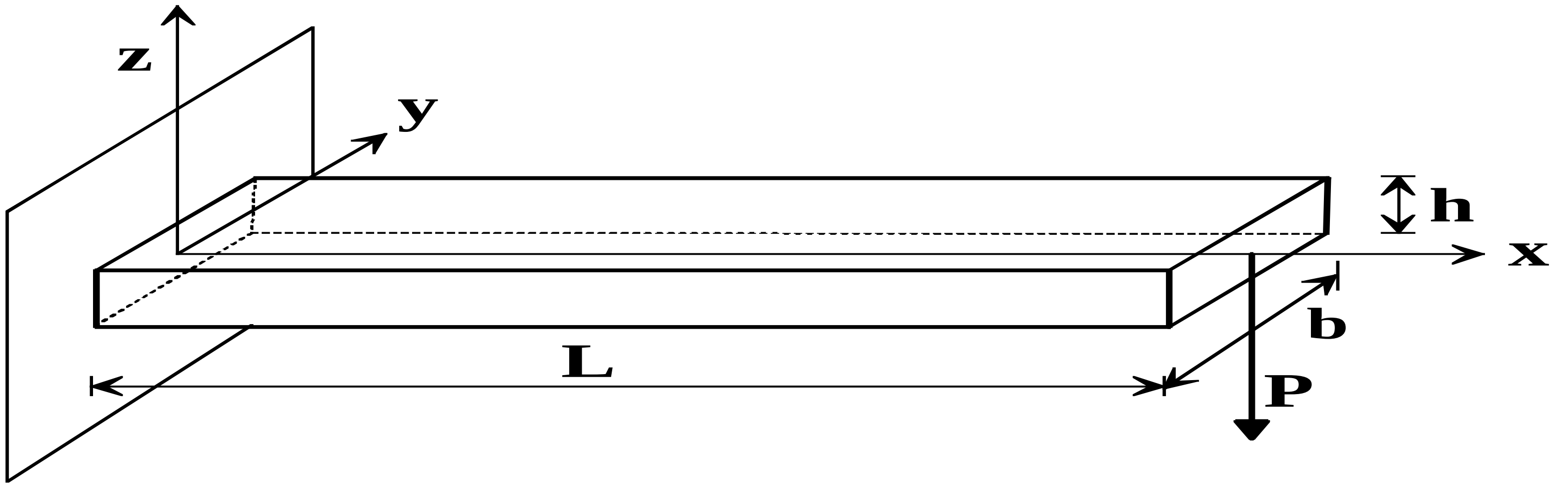
$$w = a_1 - a_2 x - a_3 y - a_4 \frac{x^2}{2} - a_5 \frac{x^3}{6} - a_6 \frac{x^2 y}{2} - a_7 \frac{x^3 y}{6} - a_8 \frac{y^2}{2} - a_9 \frac{xy^2}{2} - a_{10} \frac{y^3}{6}$$

$$-a_{11} \frac{xy^3}{6} - a_{12} \frac{xy}{2}$$

$$\beta_x = a_2 + a_4 x + a_5 \frac{x^2}{2} + a_6 xy + a_7 \frac{x^2 y}{2} + a_9 \frac{y^2}{2} + a_{11} \frac{y^3}{6} + a_{12} \frac{y}{2}$$

$$\beta_y = a_3 + a_6 \frac{x^2}{2} + a_7 \frac{x^3}{6} + a_8 y + a_9 xy + a_{10} \frac{y^2}{2} + a_{11} \frac{xy^2}{2} + a_{12} \frac{x}{2}$$

Application 3:
Cantilever plate under point load at the free end



Geometrical and material proprieties are:

L=10 , b=1.0. P=0.1., E=1.2x10^6 Poisson's ratio= v=0.0

Mesh division is 1 × 10

Vertical displacement W_{max} with the ratio (L / h)

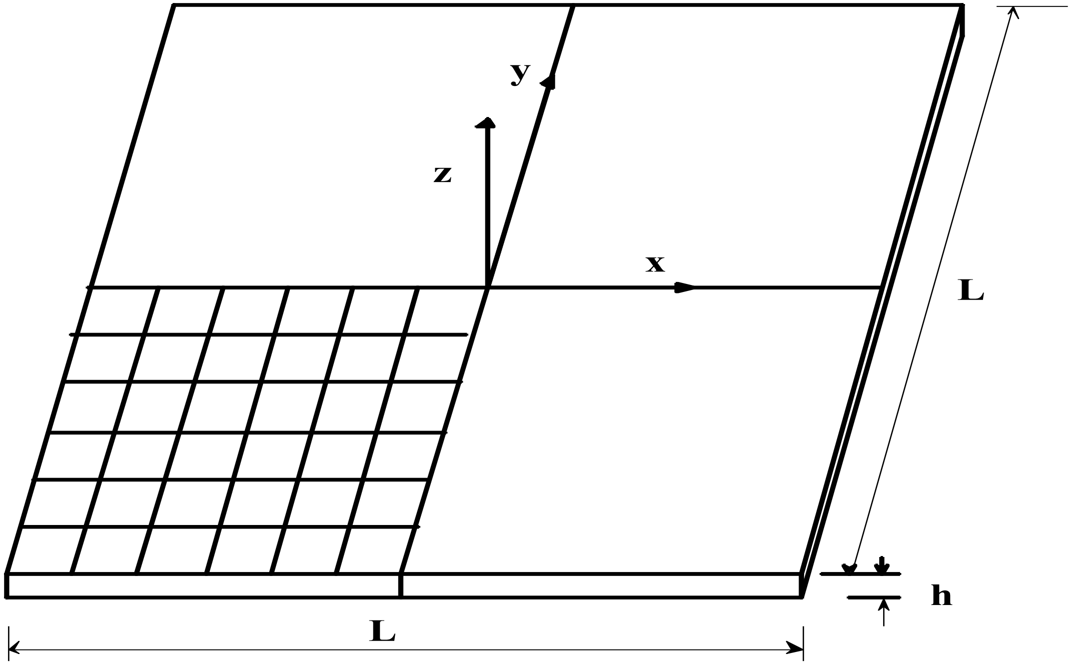
L/h	W_{max} (Normalized Values)						
	1	2	3	4	5	10	100
ACM	**0.626**	**0.864**	**0.943**	**0.972**	**0.972**	**1.042**	**1.003**
R4	**0.994**	**0.967**	**0.9583**	**0.954**	**0.906**	**0.727**	**0.023**
SBH8	**0.994**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**0.997**
SBRP	**0.994**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**0.997**
SBRPS	**0.998**	**0.987**	**0.991**	**0.995**	**0.983**	**1.006**	**1.000**
SBRPK	**0.624**	**0.858**	**0.937**	**0.968**	**0.967**	**1.009**	**1.000**
Analytical Solution	**5.33E-07**	**3.10E-06**	**9.60E-06**	**2.20E-05**	**4.30E-05**	**3.30E-04**	**0.333**

***Comments*:**

-The results obtained with SBRPK element be in good agreement with the analytical solution for L/h =100 (very thin plate).

- For L/h<10, the present element is very accurate, although it doesn't take in account the shear transverse effect.

Application 4: Simply supported and clamped Square Plate under point and distributed load), Aspect ratios L/h=100.



Geometrical and material proprieties:

L=20, h=0.2, P=1, q=1, v=0.3 and E=$10^{6\,,\,0}$

D: is the material rigidity of the plate;

***q* and *P* are the type of forces.**

The 10 × 10 mesh for the (1/4 plate)

D1= $$\left[\frac{PL^2}{D}10^{-4}\right]$$

D2= $$\frac{qL^4}{D}10^{-4}$$

Case A /Simply supported plate with central point load

Mesh	W_{max} (Normalized value)				
	ACM	SBH8	SBRP	SBRPS	SBRPK
2x2	0.934	0.220	0.157	0.147	0.800
4x4	0.980	0.7381	0.728	0.721	0.936
8x8	0.993	0.962	0.962	0.877	0.980
10x10	0.995	0.980	0.980	1.016	0.986
Analytic x D1	116.0 (1.00)				

Case B / Clamped plate with central point load

Mesh	W_{max} (Normalized value)				
	ACM	SBH8	SBRP	SBRPS	SBRPK
2x2	0.904	0.019	0.019	0.027	0.904
4x4	0.963	0.432	0.429	0.449	0.943
8x8	0.987	0.914	0.913	0.873	0.963
10x10	0.990	0.955	0.955	0.958	0.974
Analytic x D_1	56.0 (1.00)				

Case C / Simply supported plate with distributed load

Mesh	W_{max} (Normalized value)			
	ACM	SBH8	SBRP	SBRPK
2x2	0.933	0.217	0.165	0.856
4x4	0.983	0.768	0.766	0.970
8x8	0.995	0.975	0.975	0.993
10x10	0.997	0.988	0.988	0.995
Analytic x D_2	40.62 (1.00)			

Cased D / Clamped plate with distributed load

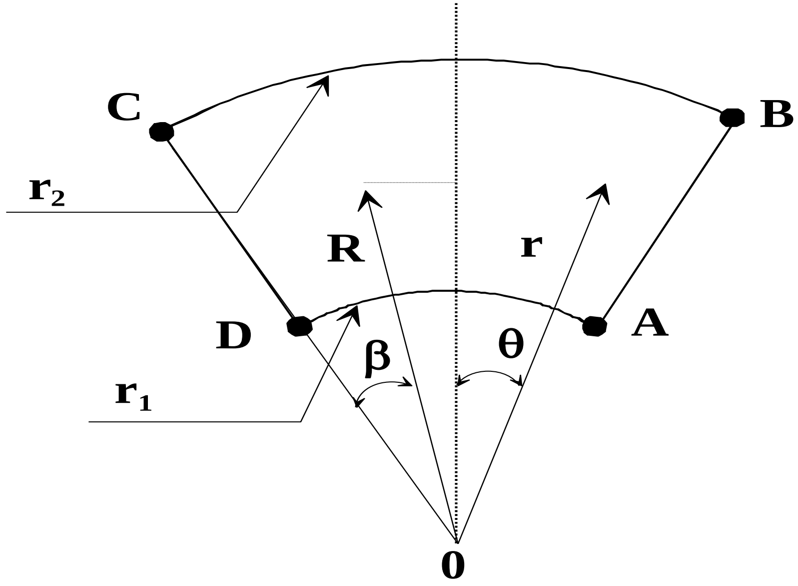
Mesh	W_{max} (Normalized value)			
	ACM	SBH8	SBRP	SBRPK
2x2	0.886	0.021	0.0214	0.825
4x4	0.965	0.446	0.443	0.886
8x8	0.987	0.936	0.935	0.964
10x10	0.990	0.972	0.972	0.976
Analytic x D_2	12.6 (1.00)			

Comments:

- The rate convergence of the developed element SBRPK is very high for all cases compared to the same strain based elements.

- The developed element has successfully handled the bending ***thin plate.***

Strain Based Sector Element “SBSPK” (Kirchhoff Theory)

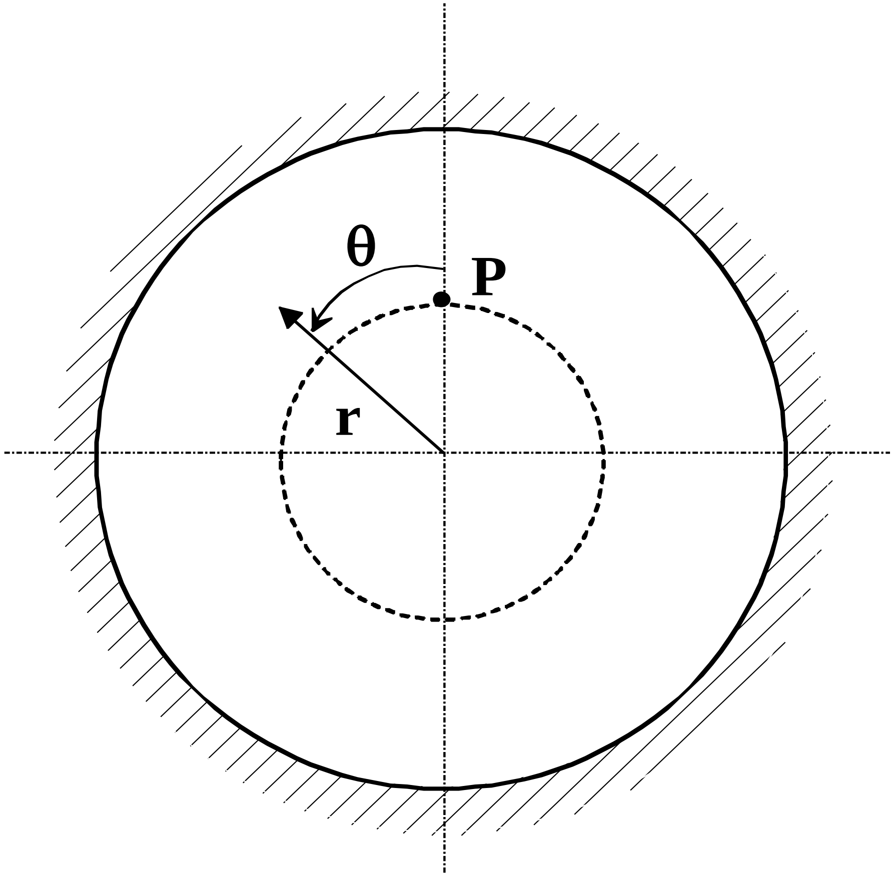


$$w = a_1 - a_2 r - a_3\theta - a_4\frac{r^2}{2} - a_5\frac{r^3}{6} - a_6\frac{r^2\theta}{2} - a_7\frac{r^3\theta}{6} - a_8\frac{\theta^2}{2} - a_9\frac{r\theta^2}{2} - a_{10}\frac{\theta^3}{6} - a_{11}\frac{r\theta^3}{6} - a_{12}\frac{r\theta}{2}$$

$$\beta_r = a_2 + a_4 r + a_5\frac{r^2}{2} + a_6 r\theta + a_7\frac{r^2\theta}{2} + a_9\frac{\theta^2}{2} + a_{11}\frac{\theta^3}{6} + a_{12}\frac{\theta}{2}$$

$$\beta_\theta = a_3 + a_6\frac{r^2}{2} + a_7\frac{r^3}{6} + a_8\theta + a_9 r\theta + a_{10}\frac{\theta^2}{2} + a_{11}\frac{r\theta^2}{2} + a_{12}\frac{r}{2}$$

Application 5: Circular plate subjected to a point load



Geometrical and material proprieties:

Clamped plate around the outer boundary (R = a),

Concentrated load P = 0.1N at r = b

h = 0,2 mm , R= 100 mm, r =50 mm, E=2x10^5 , ν = 0,3.

Lateral displacement Wmax for circular plate subjected to a point load

Mesh	Wmax (SBSPK)
1x1	0.00114
2x2	0.00180
3x3	0.00471
4x4	0.00837
4x5	0.01133
Analytical solution	0.01119

Convergence error of the lateral displacement under the applied point load

Convergence error of the lateral displacement Under the applied point load

Element	Mesh	Number of degrees of freedom	Lateral displacement	Analytical solution	Error
SBSPK	**4x5**	**30**	**0.01133**	**0.01119**	**1.25**
Olson's element	20x8	523	0.01099	0.01119	1.78

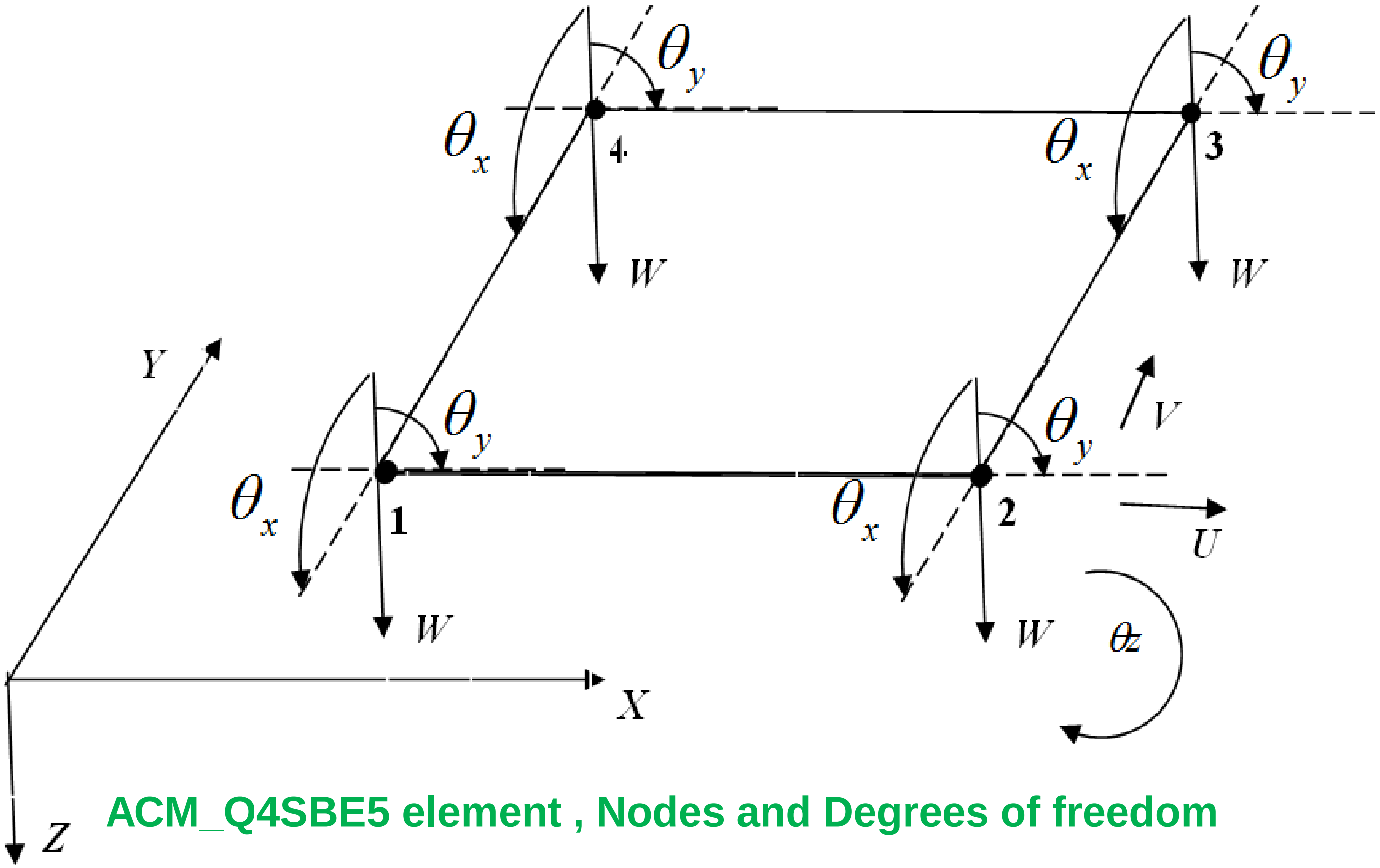
-Comments

The Strain-based element "SBSPK" is proposed for the analysis of circular thin plate bending problems (with opening)

- Only small numbers of elements, good results are obtained with small numbers of elements

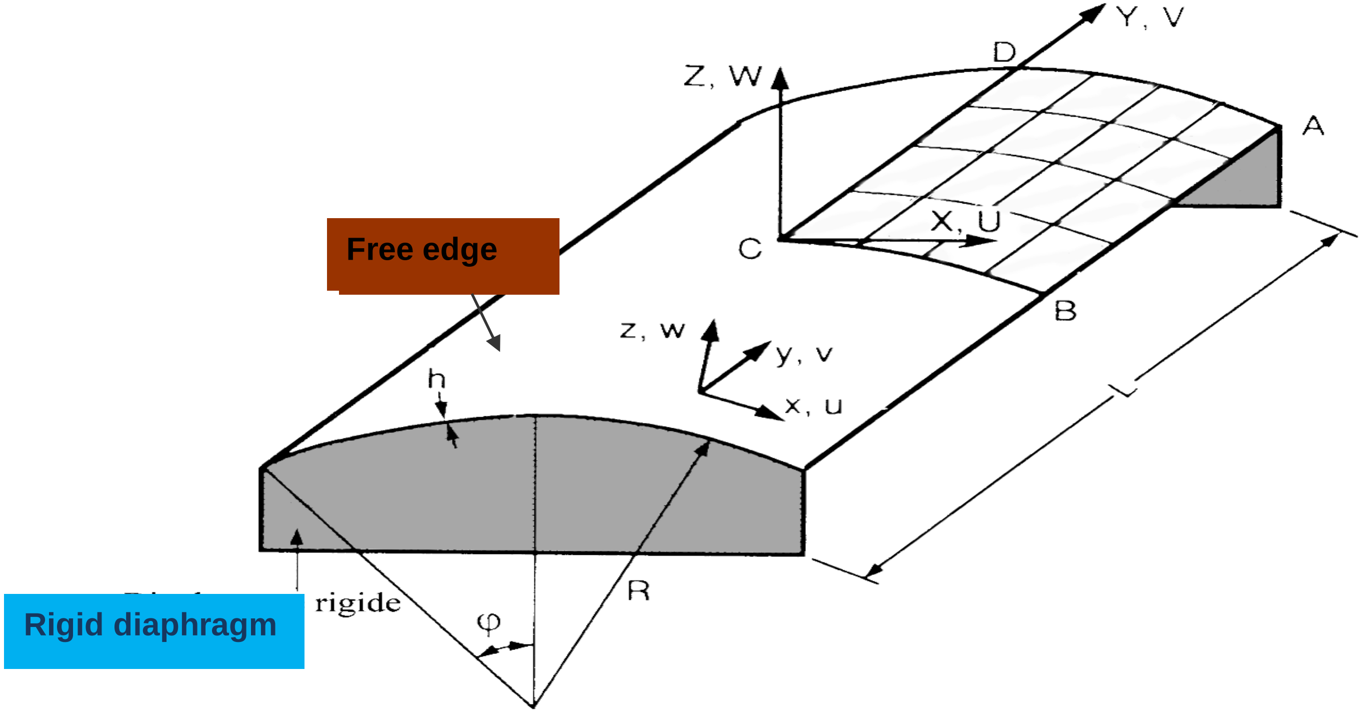
-These make the model very suitable for several civil engineering applications.

Flat Shell Element ACM_Q4SBE5



ACM_Q4SBE5 element , Nodes and Degrees of freedom

Application 6: Scordelis-Lo Roof



Free edge

Rigid diaphragm

Data: $L = 6$ m ; $R = 3$ m ; $h = 0{,}03$ m ; $\varphi = 40°$, $E = 3 \times 10^{10}$ Pa ; $\nu = 0$; $f_z = -0{,}625 \times 10^4$ Pa

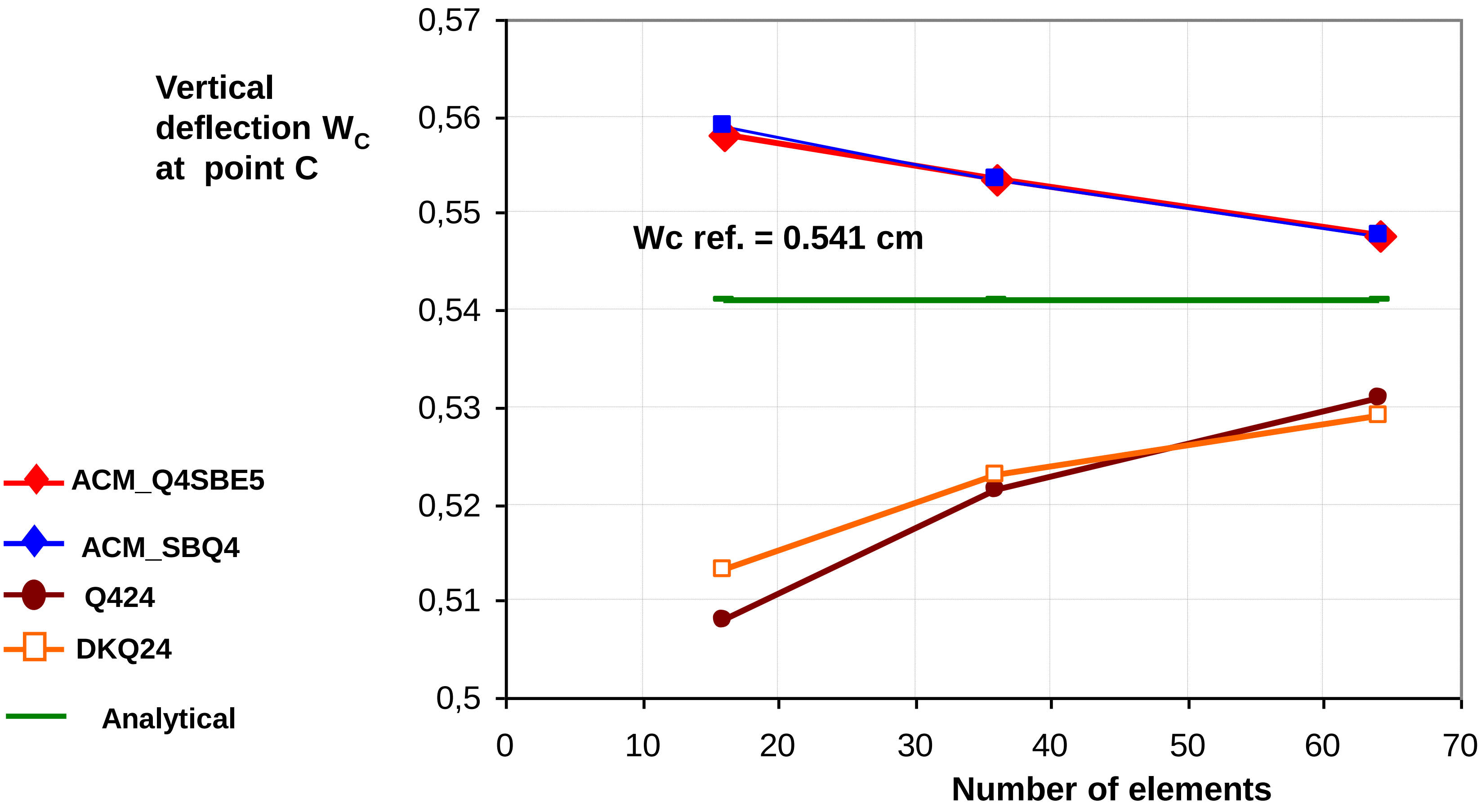
Boundary conditions: Symmetry conditions:

$U = W = \theta_Y = 0$ for AD $U = \theta_Y = \theta_Z = 0$ for CD, $V = \theta_X = \theta_Z = 0$ for CB

Reference value (Deep Shell Theory): Analytical solution (Shallow Shell theory):

$W_B = -3{,}61$ cm ; $W_C = 0{,}541$ cm , $W_B = -3{,}703$ cm ; $W_C = 0{,}525$ cm

$U_B = -1{,}965$ cm ; $V_A = -0{,}1513$ cm



Convergence curve for the deflection Wc at point C for ACM_Q4SBE5 and other quadrilateral shell elements

Conclusion

The proposed finite elements based on the strain approach **Q4SBE5, *SBMS-BH,* SBRPK, SBSPK** and **ACM_Q4SBE5:**

- ***Have the advantageous of being simple in form.***

- ***Have only the essential degrees of freedom.***

- ***Furthermore they can be used for the analysis of plane elasticity problems, plate bending and thin shell structures with good efficiency and fast convergence rate compared to reference solution and other exiting finite elements.***

Recommendation for further work

It is very useful to extend the formulated elements to:

•Application to different materials in engineering structures (composite materials, fibred reinforced polymer (FRP) composite materials etc....

- **N.L.A behaviour**
- **Dynamics behaviour and thermal effect**
- **Inclusion in FEAP, ABAQUS etc..**

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