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Dynamic Study Of Laminated Composite Plate

PRESENTATION PLAN

- INTRODUCTION
- FORMULATION OF THE (R4FSDT) BASED ON THE FIRST ORDER SHEAR DEFORMATION THEORY
- *VALIDATION* TESTS OF THE R4FSDT ELEMENT
- CONCLUSION AND PERSPECTIVES

Engineering applications « composite materials »

Figure. 1

Figure. 2

Figure. 3

Figure. 4

Figure. 5

Figure. 6

The development of modern technologies requires the use of materials **with high mechanical properties** specific to their use, but with **low densities.**

Among these materials the **laminated composite plates** which are the most used in engineering industry.

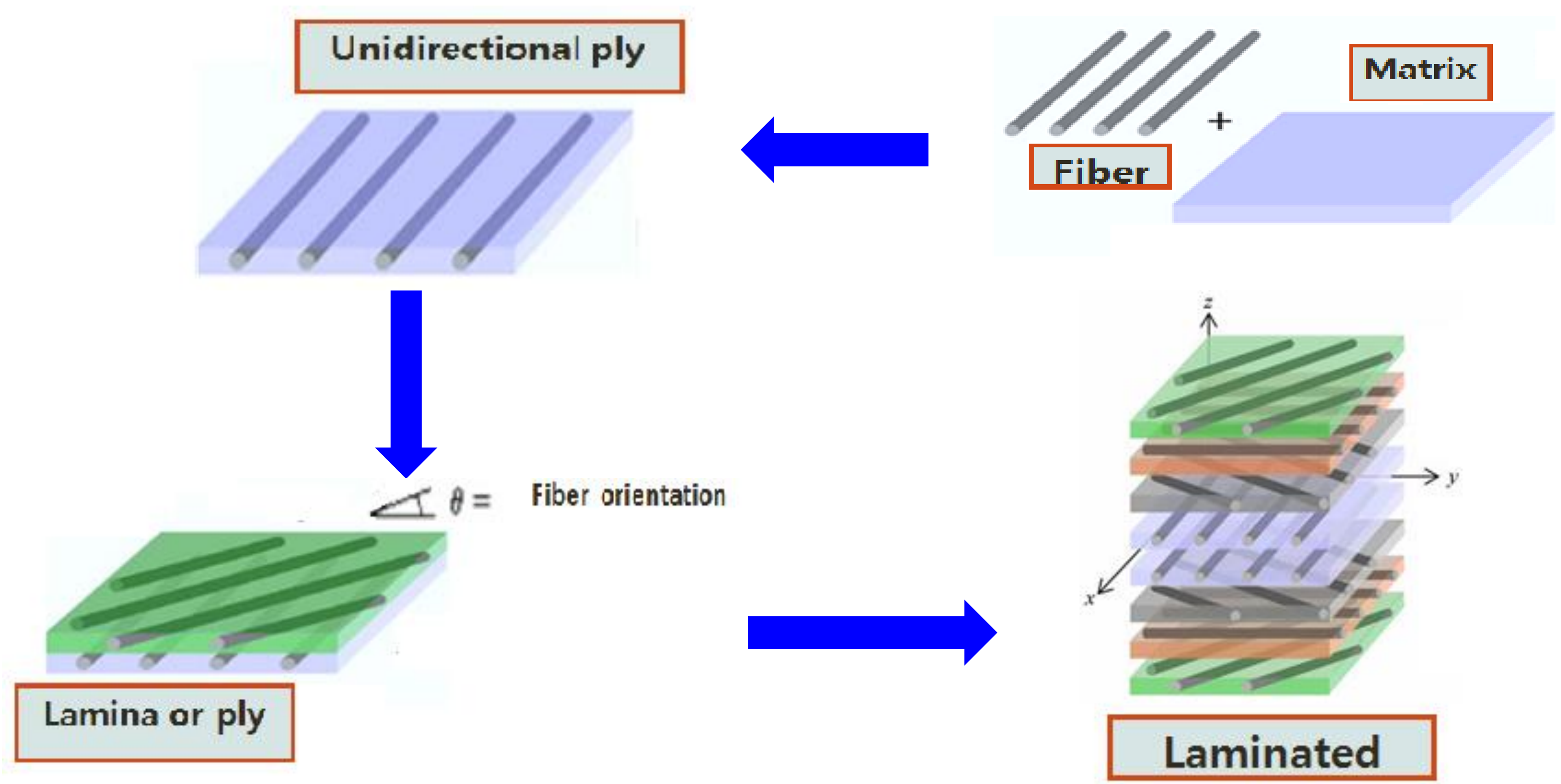


Figure. 7 **Laminated composite materials.**

For the efficient use of this type of material, it is necessary to know their structural and dynamic behaviour as well as have a precise understanding of the stress distribution and natural frequencies.

There are several theories for the study of the multilayer structures behaviour, among which we can cite :

- **Classical Lamination Plate Theory (CLPT)**
- **First order Shear Deformation Theory (FSDT)**
- **Higher order Shear Deformation Theory (HSDT)**

Gustav Kirchhoff
(1824-1887)

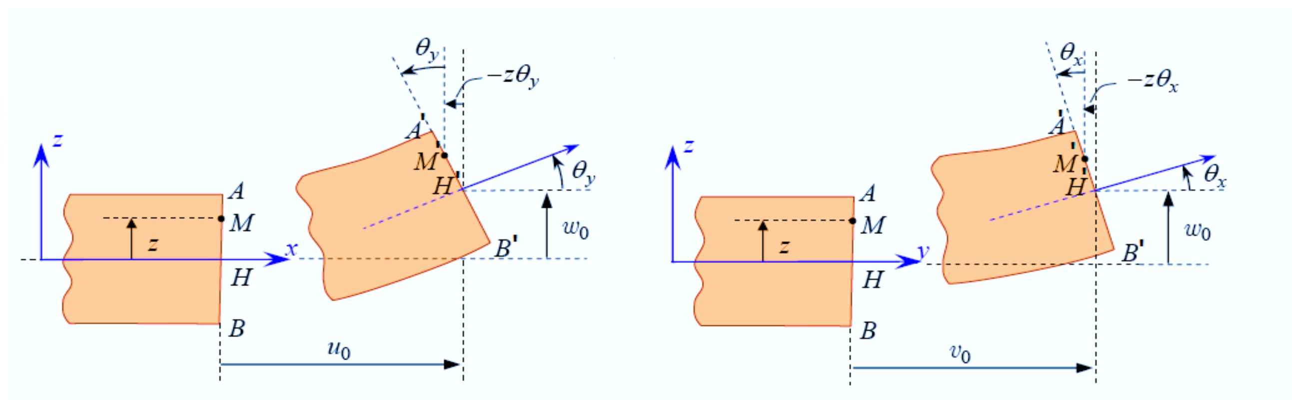
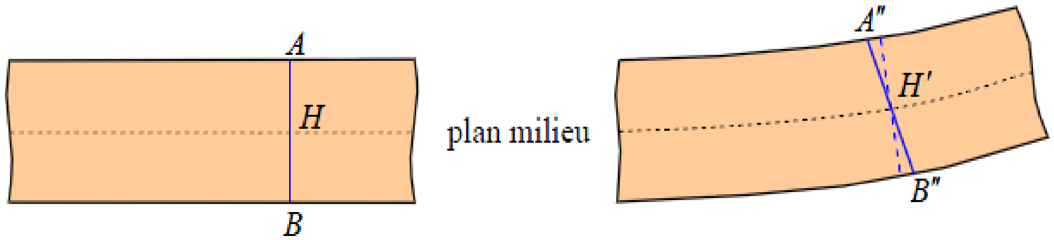


Figure. 8 **Illustration of Deformation of Classical Lamination Plate Theory**

the displacements

$$U=\begin{Bmatrix} u(x,y) \\ v(x,y) \\ w(x,y) \end{Bmatrix}=\begin{Bmatrix} u_0(x,y)-z\dfrac{\partial w_0(x,y)}{\partial x} \\ v_0(x,y)-z\dfrac{\partial w_0(x,y)}{\partial y} \\ w_0(x,y) \end{Bmatrix}$$



Eric Reissner (1913-1996)

Raymond David Mindlin (1906-1987)

Figure. 9 **Mindlin theory assumption for the shell element.**

the displacements

$$\begin{Bmatrix} u(x,y,z,t) \\ v(x,y,z,t) \\ w(x,y,z,t) \end{Bmatrix} = \begin{Bmatrix} u_0(x,y,t) + z\,\varphi_x(x,y,t) \\ v_0(x,y,t) + z\,\varphi_y(x,y,t) \\ w_0(x,y,t) \end{Bmatrix}$$

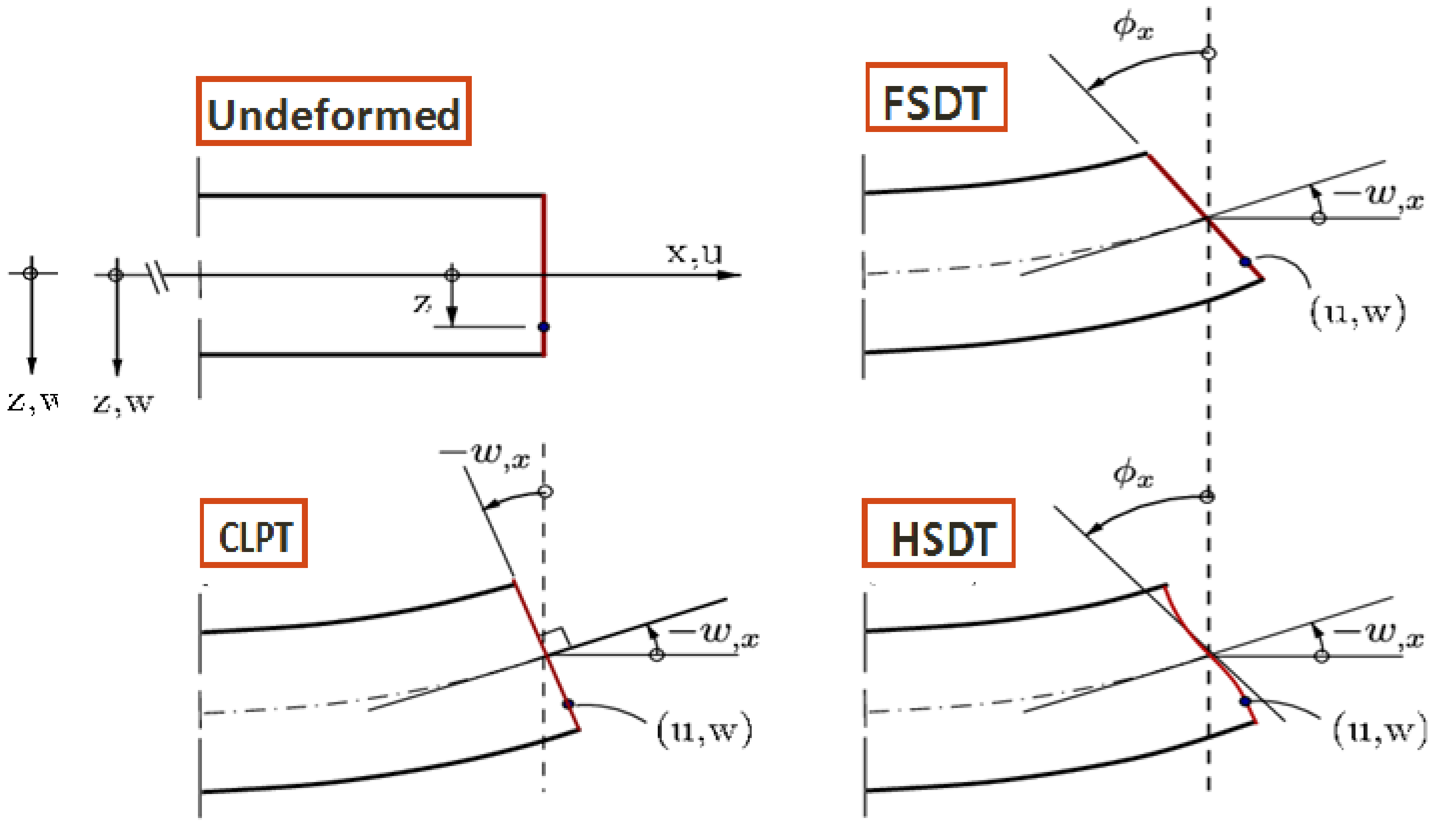


Figure. 10 **Undeformed and deformed geometries of an edge of a plate in various plate theories.**

FORMULATION OF RECTANGULAR PLATE ELEMENT BASED ON THE FIRST ORDER SHEAR DEFORMATION THEORY (R4FSDT)

STATIC ANALYSIS

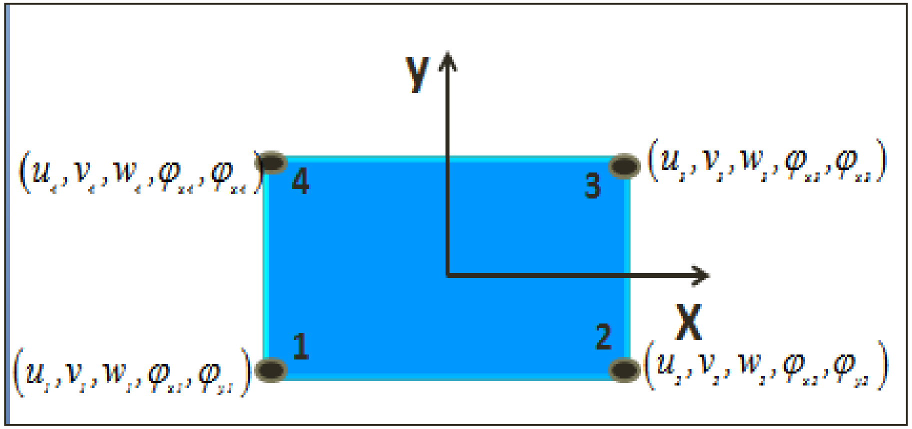


Figure .11 **Geometry of (R4FSDT) element and corresponding nodal variables**

displacement functions :

$$\begin{Bmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{Bmatrix} = \begin{Bmatrix} u_0(x,y) + z\varphi_x(x,y) \\ v_0(x,y) + z\varphi_y(x,y) \\ w_0(x,y) \end{Bmatrix}$$

➢Evaluation of the Element Stiffness Matrix :

The total potential energy of plate deformation subjected to transverse loading distributed across its surface is given by:

$$\Pi = U + W$$

▪**Potential energy of deformation is given by :**

$$U = \frac{1}{2}\int_{v} \sigma \quad \varepsilon^{T} \; dv$$

$$U = \frac{1}{2}\int_{A} \left(\{\varepsilon_m\}^T \{N\} + \{k\}^T \{M\} + \{\gamma\}^T \{Q\} \right) dA$$

$$U = \frac{1}{2}\{q\}^T \int_{A} (\{B_m\}^T [A]\{B_m\} + \{B_m\}^T [B]\{B_f\} + \{B_f\}^T [B]\{B_m\}$$
$$+ \{B_f\}^T [D]\{B_f\} + \{B_c\}^T [H]\{B_c\})\{q\} dA$$

▪The equilibrium configuration :

The equilibrium configuration is defined by the minimization of the total potential energy which means the cancellation of its first variation, namely:

$$\delta\Pi = \delta U - \delta W = 0$$

$$\int_A [\{\partial q\}^T (\{B_m\}^T [A]\{B_m\} + \{B_m\}^T [B]\{B_f\} + \{B_f\}^T [B]\{B_m\} + \{B_f\}^T [D]\{B_f\} + \{B_c\}^T [H]\{B_c\})\{q\}] dA - \{\partial q\}\{F^e\} = 0$$

The assembly of element stiffness matrices and forces vectors leads to the following equation:

$$[K]\{q\} = \{F\}$$

DYNAMICS ANALYSIS (FREE VIBRATION)

❖The dynamics equation of the laminated structure in the absence of external load is obtained by using the most generally applicable *Variational principle of the Hamilton* :

$$\delta \int_{t_1}^{t_1} (T - \Pi)\, dt = 0$$

❑kinetic energy

$$T = \frac{1}{2} \int_v \rho \frac{\partial u_i}{\partial t} \frac{\partial u_i}{\partial t} dv$$

❑Total potential energy

$$\Pi = U + \cancel{W}^{\,0}$$

The general equation of motion and for non-forced system (non-damped free vibrations) is:

$$[M]\{\ddot{q}\} + [k]\{q\} = 0$$

To obtain the vibratory Natural Frequencies amounts we have to solve the following Eigenvalue problem:

$$\left(\left[k\right]-\omega^{2}\left[M\right]\right)\left\{X\right\}=0$$

$$\det\left(\left[k\right]-\omega^{2}\left[M\right]\right)=0$$

ω ***Is the vibration natural frequency of the plate*** **(rd/s).**

$\left\{X\right\}$ ***Is the vector of global displacements (vibration modes).***

❑ Validation Tests of R4FSDT element.

Case 1 : Isotropic Square Plate

a/h	10
E	10.92 MPA
υ	0.3
G	4.2 MPA
P	1N/m²

$$W_{max} = \alpha \cdot \frac{D}{q a^4}$$

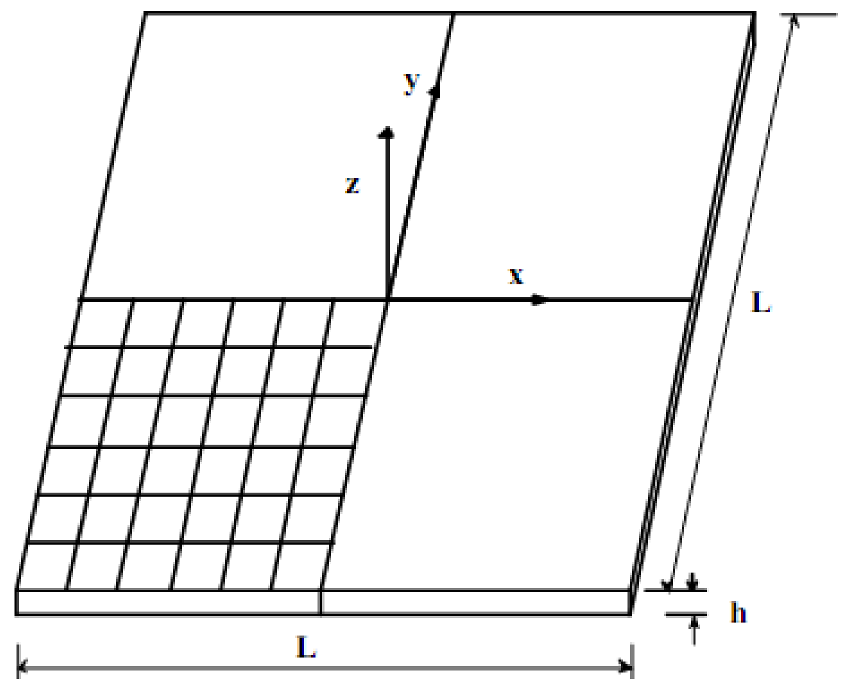


TABLE 1: The Maximum Deflection at the Centre of Simply Supported Isotropic Square Plate

Mesh	Maximum deflection	Error (%)
(2*2)	0.09	78.64
(4*4)	0.23	45.73
(6*6)	0.31	25.76
(8*8)	0.36	14.47
(10*10)	0.393	7.82
(12*12)	0.41	3.62
(14*14)	0.42	0.86
Analytical solution (Reddy)	0.42	
Numerical results (Ferreira)	0.42	

❑ Validation Tests of R4FSDT element (Dynamics analysis)

Case 1: Isotropic simply supported square plated

a/h	10
E	10.92 MPA
υ	0.3
G P	4.2 MPA $1N/m^2$

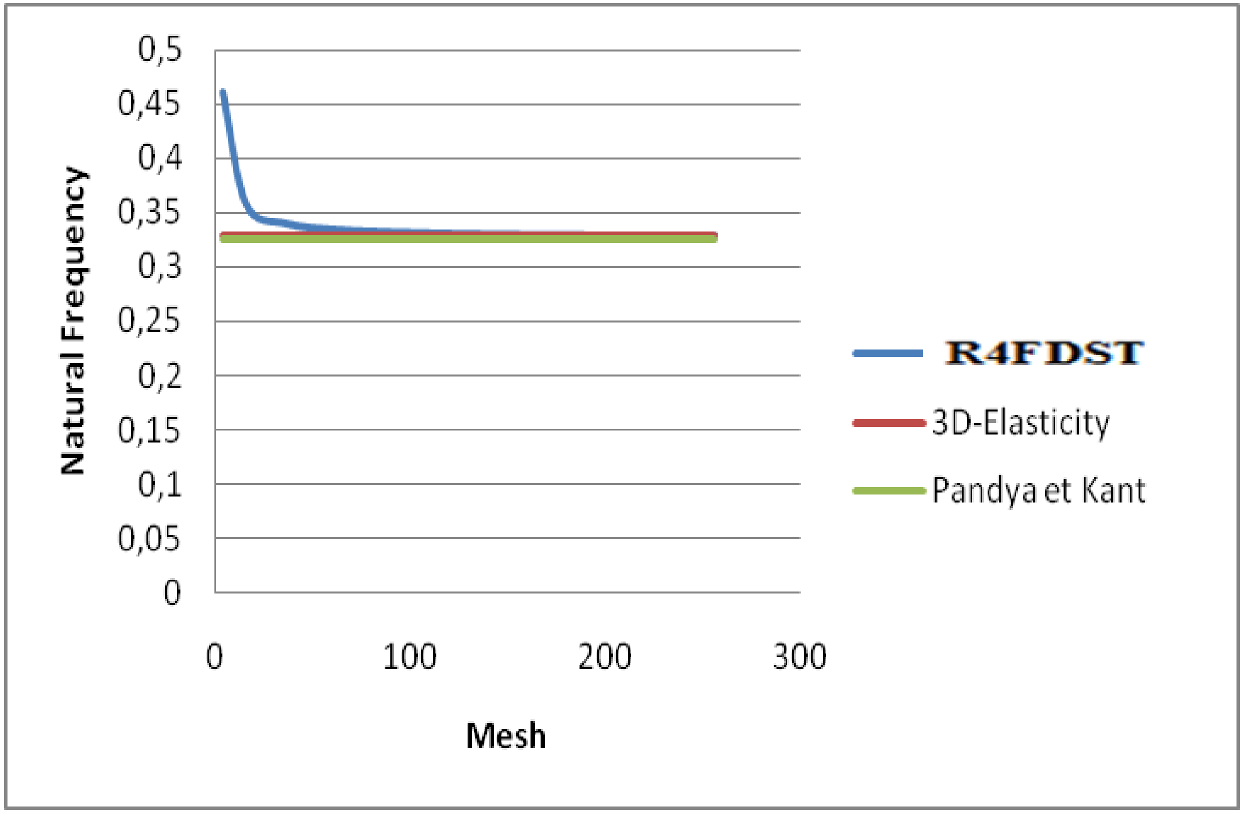
$$\overline{w} = w\, a \sqrt{\frac{\rho}{G}}$$

TABLE 2: The natural frequency of a simply supported isotropic square plate

Mesh	Natural frequency(rd/s)	Error (%)
(4*4)	1.34	44.62
(8*8)	1.03	11.72
(12*12)	0.97	5.24
(16*16)	0.95	2.94
(20*20)	0.94	1.87
(24*24)	0.94	1.32
Analytical solution (Reddy)	0.93	
Numerical results (Ferreira)	0.93	

Case 2: Laminated composite square plate.

a/h	5
fiber orientation	(0/90/0)
$\frac{E_{11}}{E_{22}}$	10
$\frac{G_{12}}{E_{22}} = \frac{G_{13}}{E_{22}}$	0.6
$\frac{G_{23}}{E_{22}}$	0.5
ν_{12}	0.25
ρ	1



$$\overline{w} = wah\sqrt{\frac{\rho}{E_{22}}}$$

Figure .12 **The natural frequency of a laminated simply supported square plate (0/90/0)**

✓The performance of the developed element based on the first order shear deformation has been examined through a comparative study on the maximum deflections and eigenvalues frequency with the analytical solution .

✓The comparison of the numerical results obtained shows the rapid convergence of the presented element.

As a perspective, we suggest expanding this study to:

- Using a more performance element based on higher order theories of transverse shearing.
- Adopting other approaches such as the layerwise approach (layerwise theory).
- Applications of the developed element to other more complex structure .
- Extend the study to the non-linear behavior of a stratified plate.

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